Exponents and Radicals

Product Rule for Exponents

$a^n \cdot a^m = a^{m+n}$

Simplify:

$2^1 \cdot 2^0$

$5^2 \cdot 5^0$

Zero Exponent

If a is any non zero real number then $a^0 = 1$.

(Note: 0^0 is not a number)

Simplify:

$2^1 \cdot 2^{-1}$

$5^2 \cdot 5^{-2}$

Negative exponents

For any nonzero number a,

$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$

Quotient Rule for exponents

$\frac{a^n}{b^m} = a^{n-m}$

Simplify the following and write the result using only positive exponents

$\frac{x^5}{x^2}$

$\frac{2\,x^{-3}\,y}{4\,x^{-5}y^3}$

Power Rules for Exponents

$(a^m)^n = a^{mn}$ $\quad (ab)^n = a^n b^n$ $\quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Simplify:

$(a^2)^{-4}$

$(a\,b)^4\,a^{-2}\,b^3$

Simplify:

$$\frac{\left(-5\,y^3\,z^4\right)^2}{10\,(y^{-4}\,z^2)^{-3}}$$

Radical Notation:

If all of the indicated roots are real numbers, then:

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m \text{ or } a^{m/n} = \sqrt[n]{a^m}$$

Simplify:

$$9^{3/2} \qquad\qquad (-8)^{2/3}$$

Simplify:

$$\frac{(x^2\,y^5)^{-1/4}}{(x^{-3}\,y^2)^{1/6}}$$

Simplifies Radical Expressions:

1. The radicand has no factors raised to a power greater than or equal to the index.
2. The radicand has no fractions.
3. No denominator contains a radical.
4. Exponents in the radicand and the index have no common factors other than 1.

Properties of Radicals:

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then:

1) $\sqrt[n]{a^n} = |a|$ when n is even.

2) $\sqrt[n]{a^n} = a$ when n is odd.

3) $\sqrt[n]{ab} = \sqrt[n]{a}\ \sqrt[n]{b}$

4) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

5) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Simplify: $\sqrt{44}$

Simplify: $\sqrt[3]{16\,x^4\,y^2}$

Simplify: $\sqrt{\frac{2\,x^3}{27}}$

Simplify: $\sqrt[3]{\frac{16\,x^4}{9}}$

Simplify: $\frac{8}{\sqrt{3}+\sqrt{7}}$